GCE AS/A LEVEL – NEW

2305U30-1

S18-2305U30-1

FURTHER MATHEMATICS – AS unit 3
FURTHER MECHANICS A

TUESDAY, 22 MAY 2018 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as 9·8ms^{-2}.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: *Sufficient working must be shown to demonstrate the* ***mathematical*** *method employed.*

1. Two objects, *A* of mass 18 kg and *B* of mass 7 kg, are moving in the same straight line on a smooth horizontal surface. Initially, they are moving with the same speed of 4 ms^{-1} and in the same direction. Object *B* collides with a vertical wall which is perpendicular to its direction of motion and rebounds with a speed of 3 ms^{-1}. Subsequently, the two objects *A* and *B* collide directly. The coefficient of restitution between the two objects is $\frac{5}{7}$.

(a) Find the coefficient of restitution between *B* and the wall. [1]

(b) Determine the speed of *A* and the speed of *B* immediately after the two objects collide. [7]

(c) Calculate the impulse exerted by *A* on *B* due to the collision and clearly state its units. [2]

(d) Find the loss in energy due to the collision between *A* and *B*. [2]

(e) State the direction of motion of *A* relative to the wall after the collision with *B*. [1]

2. A car of mass 750 kg is moving on a slope inclined at an angle θ to the horizontal, where $\sin\theta$ = 0·1. When the car's engine is working at a constant power P W, the car can travel at maximum speeds of 14 ms^{-1} up the slope and 28 ms^{-1} down the slope. In each case, the resistance to motion experienced by the car is proportional to the square of its speed. Find the value of P and determine the resistance to the motion of the car when its speed is 10·5 ms^{-1}. [10]

3. A light elastic string of natural length 1·5 m and modulus of elasticity 490 N has one end attached to a fixed point *A* and the other end attached to a particle *P* of mass 30 kg. Initially, *P* is held at rest vertically below *A* such that the distance *AP* is 0·6 m. It is then allowed to fall vertically.

(a) Calculate the distance *AP* when *P* is instantaneously at rest for the first time, giving your answer correct to 2 decimal places. [8]

(b) Estimate the distance *AP* when *P* is instantaneously at rest for the second time and clearly state one assumption that you have made in making your estimate. [2]

4. The position vector $\mathbf{x}$ metres at time t seconds of an object of mass 3 kg may be modelled by

$$\mathbf{x} = 3\sin t\,\mathbf{i} - 4\cos 2t\,\mathbf{j} + 5\sin t\,\mathbf{k}.$$

(a) Find an expression for the velocity vector $\mathbf{v}$ ms^{-1} at time t seconds and determine the least value of t when the object is instantaneously at rest. [7]

(b) Write down the momentum vector at time t seconds. [1]

(c) Find, in vector form, an expression for the force acting on the object at time t seconds. [3]

5. A particle *P*, of mass m kg, is attached to one end of a light inextensible string of length l m. The other end of the string is attached to a fixed point *O*. Initially, *P* is held at rest with the string just taut and making an angle of 60° with the downward vertical. It is then given a velocity u ms^{-1} perpendicular to the string in a downward direction.

(a) (i) When the string makes an angle θ with the downward vertical, the velocity of the particle is v and the tension in the string is T. Find an expression for T in terms of m, l, u^2 and θ.

(ii) Given that *P* describes complete circles in the subsequent motion, show that $u^2 > 4lg$. [10]

(b) Given that now $u^2 = 3lg$, find the position of the string when circular motion ceases. Briefly describe the motion of *P* after circular motion has ceased. [3]

(c) The string is replaced by a light rigid rod. Given that *P* describes complete circles in the subsequent motion, show that $u^2 > klg$, where k is to be determined. [2]

6. A vehicle of mass 1200 kg is moving with a constant speed of 40 ms^{-1} around a horizontal circular path which is on a test track banked at an angle of 60° to the horizontal. There is no tendency to sideslip at this speed. The vehicle is modelled as a particle.

(a) Calculate the normal reaction of the track on the vehicle. [3]

(b) Determine

(i) the radius of the circular path,

(ii) the angular speed of the vehicle and clearly state its units. [6]

(c) What further assumption have you made in your solution to *(b)*? Briefly explain what effect this assumption has on the radius of the circular path. [2]

END OF PAPER

